AERODYNAMICS

FUNDAMENTAL PRINCIPLES

ACCELERATION:

The below equation tells that the operator for total differential wrt time, D/Dt in a convective field is related to the partial differential as:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

1. The total differential D/Dt is known as the **material or substantial derivative** with respect to time.
2. The first term ρ/ρt in the right hand side is known as **temporal or local derivative** which expresses the rate of change with time, at a fixed position.
3. The last three terms in the right hand side are together known as **convective** which represents the time rate of change due to change in position in the field.

Material or substantial acceleration = (temporal or local acceleration) + (convective acceleration)

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

NOTE:

In a steady flow, the local acceleration is zero, since the velocity at any point is invariant with time.

In a uniform flow, on the other hand , the convective acceleration is zero, since the velocity components are not the functions of space coordinates.

In a steady and uniform flow, both the local and convective acceleration vanish and hence there exists no material acceleration.

ROTATION (ω):

It is the angle of velocity of particle about its centre of mass.

Rotation vector, $\overline{\omega} = \omega_x\hat{\imath} + \omega_y\hat{\jmath} + \omega_z\hat{k}$

Where,

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

VORTICITY(ξ):

It is the spinning property of a rotating mass multitude.

$$\xi = \nabla \times \bar{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$= 2\,[\omega_x \hat{\imath} + \omega_y \hat{\jmath} + \omega_z \hat{k}\ ]$$

$$\xi = \nabla \times \bar{V} = 2\overline{\omega}$$

Vorticity = 2 $\times$ Rotation

Note:

- **If curl of a velocity vector is zero, i.e. vorticity is zero, than the flow is said to be irrotational, this implies that the fluid elements have no angular velocity; rather, their motion through space is a pure translation.**
- **For 2D flow, $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$, is the condition of irrotationality.**

CIRCULATION:

It is defined as the strength of the vortex. It is the line integral around a closed curve of the velocity field.

$$\mathrm{r} = \oint \bar{V}.\overline{ds} = \oiint (\nabla \times \bar{V}).\overline{ds}$$

$$\frac{\partial \mathrm{r}}{\partial s} = (\nabla \times \bar{V}).\hat{n} = \xi$$

NOTE:

- **For irrotational flow, circulation (r) = 0**
- **Particles of the flow must rotate about its own mass centre to be called as rotational flow.**

CONSERVATION EQUATIONS

CONTINUITY EQUATION

Integral form:

$$\frac{\partial}{\partial t} \oiiint \rho dV + \oiint \rho \bar{V}.\overline{ds} = 0$$

Ist Term = Rate of decrement of mass within the control volume

2nd Term = Net mass flow rate through cross section.

For steady flow,

$$\oiint \rho \bar{V}.\overline{ds} = 0$$

For steady, incompressible flow,

$$\oiint \bar{V}.\overline{ds} = 0$$

Differential form:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho.\bar{V}) = 0$$

For steady flow,

$$\nabla(\rho.\bar{V}) = 0$$

$$\text{i.e. } \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

For incompressible flow,

$$\nabla.\bar{V} = 0$$

$$\text{i.e. } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Note : Above equation is valid for both steady and unsteady flows.

MOMENTUM EQUATION

Integral form:

$$\frac{\partial}{\partial t}\oiiint \rho\bar{V}dV + \oiint(\rho\bar{V}.\overline{ds})\bar{V} = -\oiint P\overline{ds} + F_{body\ forces} + F_{Viscous}$$

Differential form:

$$\frac{\partial(\rho u)}{\partial t} + \nabla(\rho u\bar{V}) = -\frac{\partial P}{\partial x} + F_{body\ forces} + F_{Viscous}$$ x –Direction

$$\frac{\partial(\rho v)}{\partial t} + \nabla(\rho v\bar{V}) = -\frac{\partial P}{\partial y} + F_{body\ forces} + F_{Viscous}$$ y –Direction

$$\frac{\partial(\rho w)}{\partial t} + \nabla(\rho w\bar{V}) = -\frac{\partial P}{\partial z} + F_{body\ forces} + F_{Viscous}$$ z –Direction

For steady, inviscid and no body forces,

$\nabla(\rho u\bar{V}) = -\frac{\partial P}{\partial x}$ x –Direction

$\nabla(\rho v\bar{V}) = -\frac{\partial P}{\partial y}$ y –Direction

$\nabla(\rho w\bar{V}) = -\frac{\partial P}{\partial z}$ z –Direction

NOTE:

- **In a fluid mechanics problem, there are 4 unknowns (u, v, w and P). The four set of equations required to solve for these 4 unknowns are 1 continuity equation and 3 momentum equation.**
- **The Momentum equations for an inviscid flow are called the Euler equation.**
- **The Momentum equations for a viscous flow are called the Navier-Stokes equations.**